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On the brane coupling of Unified orbifolds with gauge interactions in the bulk

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Abstract

In the on-shell formulation of $D = 5$, $N = 2$ supergravity, compactified on S^1/Z_2, we extend the results of Mirabelli and Peskin describing the interaction of the bulk fields with matter which is assumed to be confined on the brane. The novel characteristics of this approach are : Propagation of both gravity and gauge fields in the bulk, which offers an alternative for a unified description of models in extra dimensions and use of the on-shell formulation avoiding the complexity of off-shell schemes which involve numerous auxiliary fields. We also allow for nontrivial superpotential interactions of the chiral matter fields.

The method we employ uses the Nöther procedure and our findings are useful for building models advocating propagation of the gauge degrees of freedom in the bulk, in addition to gravity.

1 Introduction

It is well established that the Standard Model (SM) describes succesfully all particle interactions at low energies. On the other hand it is understood that SM is an effective theory. At high energies, description of the elementary particle interactions demands a generalization of the SM. Assuming a unified description in terms of a renormalizable field theory, up to very high energies lead to favorable generalization namely GUT theories [1], among which supersymmetric GUTs [2] play a central rôle. Consistent inclusion of gravity dictates that these generalizations should be effective descriptions of a more fundamental underlying theory. String Theory [3] is the most prominent candidate for this aim. Indeed from the 10 dimensional field theory, which is the effective point limit of the String Theory we can get, by suitable compactifications of the extra dimensions, consistent four-dimensional models compatible with the SM [4]. Along these lines it has been conjectured that one or two dimensions may be compactified at different scales, lower from the remaining ones [5]. Also after the developments concerning the duality symmetries of String Theory and in the framework of M-Theory [6, 7], the idea that our world may be a brane embedded in a higher dimensional space has recently attracted much interest and has been studied intensively [8–12]. Besides the original compactifications new possibilities have been proposed [13]. It has been also recognized that String/M theory may lead to brane-world models in which one of the extra dimensions can be even non-compact [14, 15]. In all these models the four-dimensional world is a brane, on which the matter fields live, while gravity, and in some interesting cases the gauge and the Higgs fields, propagate also in the transverse extra dimensions of the bulk space.

In the majority of the cases studied, in an attempt to build realistic models, the bulk is a five-dimensional space [16]. In these models the corresponding backgrounds may be of Minkowski or Anti-de-Sitter type. Effects of the above consideration in specific GUT models have been also considered. The assumed background for these models is of Minkowski type and questions regarding the unification and supersymmetry breaking scales have been addressed to. In this direction assuming the fifth dimension very large, of the TeV scale, non supersymmetric extensions of the SM even without the need of unification have been considered [17–22]. On the other hand models embedding the SM in an Anti-de-Sitter five dimensional space have been also discussed [23, 24].

In view of the aforementioned developments the study of the five dimensional supergravities has been revived [25–28]. This is quite natural since after all gravity is in the center of all these attempts and it is legitimate to assume that we have to treat the

fifth dimension before going to a "flat limit". In these recent considerations of the five-dimensional supergravity no specific model based on a particular gauge group has been introduced so far. Also the interaction of the brane multiplets with the bulk gauge fields, essential for Supersymmetry and the transmition of its breaking, [16], has not been studied in the context of D=5, N=2 supergravity models in which gauge fields are allowed to propagate in the bulk in addition to gravity. In this note we undertake this in the on shell formulation of five - dimensional supergravity.

2 Setting the model

We consider a five-dimensional Yang-Mills supergravity model. The field content of the model is [25]

$$\{e_{\tilde{\mu}}^{\tilde{m}}, \Psi_{\tilde{\mu}}^{i}, A_{\tilde{\mu}}^{I}, \lambda^{ia}, \phi^{x}\} \tag{1}$$

where $\tilde{\mu} = (\mu, 5)$ are curved and $\tilde{m} = (m, \dot{5})$ are flat five-dimensional indices, with μ, m their corresponding four dimensional indices. The remaining indices are $I = 0, 1, \ldots, n$, $a = 1, \ldots, n$ and $x = 1, \ldots, n$. The supergravity multiplet consists of the fünfbein $e_{\tilde{\mu}}^{\tilde{m}}$, two gravitini $\Psi_{\tilde{\mu}}^{i}$ and the graviphoton $A_{\tilde{\mu}}^{0}$, where $i = 1, 2$ is the symplectic $SU(2)_R$ index. Moreover, there exist n vector multiplets, counting the Yang-Mills fields ($A_{\tilde{\mu}}^{a}$). The spinor and the scalar fields included in the vector multiplets are collectively denoted by λ^{ia}, ϕ^{x} respectively. The indices a, x are flat and curved indices respectively of the n-dimensional manifold $\mathcal{M}$ parametrized by the scalar fields. This manifold is embedded in an $(n+1)$-dimensional space and it is determined by the cubic constraint

$$C_{IJK} h^I h^J h^K = 1. \tag{2}$$

h^I are functions of the scalar fields defining the embedding of the manifold $\mathcal{M}$. C_{IJK} are constants symmetric in the three indices.

The assumption that the gauge interactions propagate in the five-dimensional bulk while only the matter fields are localized on the branes is implemented as follows. We consider five-dimensional vector fields $A_{\tilde{\mu}}^{a}$, and we perform the S_1/Z_2 orbifold by assigning Z_2-even parity to the four dimensional part of the vectors A_{μ}^{a} and Z_2-odd parity to the fifth component A_5^a. The five-dimensional Yang-Mills Einstein Lagrangian has to be even for the parity assignments to be consistent. The only term that may cause problem is the Chern-Simons term

$$\frac{\epsilon^{\tilde{\mu}\tilde{\nu}\tilde{\rho}\tilde{\sigma}\tilde{\lambda}}}{6\sqrt{6}} C_{IJK} \Big\{ F_{\tilde{\mu}\tilde{\nu}}^{I} F_{\tilde{\rho}\tilde{\sigma}}^{J} A_{\tilde{\lambda}}^{K} + \frac{3}{2} g F_{\tilde{\mu}\tilde{\nu}}^{I} A_{\tilde{\rho}}^{J} (f_{LF}^{K} A_{\tilde{\sigma}}^{L} A_{\tilde{\lambda}}^{F}) + \frac{3}{5} g^2 (f_{GH}^{J} A_{\tilde{\nu}}^{G} A_{\tilde{\rho}}^{H})(f_{LF}^{K} A_{\tilde{\sigma}}^{L} A_{\tilde{\lambda}}^{F}) A_{\tilde{\mu}}^{I} \Big\} \tag{3}$$

where f_{IJ}^K above are the structure constants. In particular f_{ab}^c are the structure constants of the non-abelian gauge group and $f_{IJ}^K = 0$ if any one of the indices is 0. g is the gauge coupling constant. This term is in general odd for the assignment given above. Nevertheless if we chose the coefficients of the cubic constraint to be in the canonical basis

$$C_{000} = 1, \quad C_{0ab} = -\frac{1}{2}\delta_{ab}, \quad C_{00a} = 0, \quad C_{abc} = \text{arbitrary} \tag{4}$$

and give Z_2-odd parity to the four dimensional part of the graviphoton, A_μ^0, and Z_2-even parity to its fifth component A_5^0 then the Chern-Simons term becomes even and the parity assignment is consistent with the choice $C_{abc} = 0$ which we assume in the following. For example one of the terms in (3) is $\sim \epsilon^{\mu\nu\rho\sigma 5} C_{IJK} F_{\mu\nu}^I F_{\rho\sigma}^J A_5^K$ and if we take A_μ^a to be Z_2-even and A_5^a to be Z_2-odd, obviously C_{abc} have to vanish. Also since in the canonical basis we are enforced to take $C_{0ab} = -\frac{1}{2}\delta_{ab}$ the corresponding Chern-Simons term is even with A_μ^0 Z_2-odd and A_5^0 Z_2-even. The remaining terms in the expansion of (3) may be treated accordingly.

Considering now the full spectrum, the Z_2-even fields are

$$e_\mu^m, \quad e_5^{\dot{5}}, \quad \Psi_\mu^1, \quad \Psi_5^2, \quad A_5^0, \quad A_\mu^a, \quad \lambda^{1a},$$

while the Z_2-odd fields are

$$e_\mu^{\dot{5}}, \quad e_5^m, \quad \Psi_\mu^2, \quad \Psi_5^1, \quad A_\mu^0, \quad A_5^a, \quad \lambda^{2a}, \quad \phi^x.$$

These parity assignments complete the S_1/Z_2 orbifold. All the fields of the abovementioned spectrum propagate in the bulk. Only the even fields propagate on the two branes located at $x^5 = 0$ and $x^5 = \pi R$, that is the fixed points of the Z_2 transformation.

The spectrum of the even fields respects four-dimensional N=1 supersymmetry. The model is supplied by chiral multiplets localized on the branes. The coupling of these multiplets to the bulk fields is determined by the N=1 Supersymmetry invariance of the total action.

The bulk Lagrangian is [26]

$$\begin{aligned} \mathcal{L}_0/e^{(5)} &= -\frac{1}{2}R^{(5)} + \frac{i}{2}\bar{\Psi}_{i\tilde{\mu}}\gamma^{\tilde{\mu}\tilde{\nu}\tilde{\rho}}\nabla_{\tilde{\nu}}\Psi^i_{\tilde{\rho}} - \frac{1}{4}\mathring{a}_{IJ}F^I_{\tilde{\mu}\tilde{\nu}}F^{I\,\tilde{\mu}\tilde{\nu}} - \frac{1}{2}g_{xy}(\mathcal{D}_{\tilde{\mu}}\phi^x)(\mathcal{D}^{\tilde{\mu}}\phi^y) \\ &+ \ \mathrm{Fermion} \ + \ \mathrm{Chern-Simons\ \ terms} \end{aligned} \tag{5}$$

The tensor $\mathring{a}_{IJ}$, appearing in the kinetic terms of the gauge fields, is the restriction of the metric of the $(n+1)$- dimensional space on the n-dimensional manifold of the scalar fields given by

$$\mathring{a}_{IJ} = -2C_{IJK}h^K + 3h_I h_J \ , \tag{6}$$

where $h_I = C_{IJK} h^J h^K = \mathring{a}_{IJ} h^J$ and $g_{xy} = h^I_x h^J_y \mathring{a}_{IJ}$ is the metric of the n-dimensional manifold $\mathcal{M}$. In these equations $h^I_x = -\sqrt{\frac{3}{2}} h^I{}_{,x}$ and $h_{Ix} = \sqrt{\frac{3}{2}} h_{I,x}$. Note also that the following relations hold

$$h^I h_I = 1, \quad h^I_x h_I = h^I h_{Ix} = 0 \ . \tag{7}$$

With the parity assignments we have adopted, h^0 is even, while $h^x = \phi^x$ are odd. Furthermore on the fixed points where the odd quantities vanish, $h^0 = 1$. Analogous relations hold for the h_I's.

3 The Supersymmetry Transformations

Recalling the linearized supersymmetry transformations of the bulk fields

$$\begin{aligned}
\delta e^{\tilde{m}}_{\tilde{\mu}} &= i \bar{\epsilon}_i \gamma^{\tilde{m}} \Psi^i_{\tilde{\mu}} \\
\delta \Psi^i_{\tilde{\mu}} &= 2 \nabla_{\tilde{\mu}}(\omega) \epsilon^i - \frac{h_I}{2\sqrt{6}} \gamma^{\tilde{\nu}\tilde{\rho}}_{\tilde{\mu}} F^I_{\tilde{\nu}\tilde{\rho}} \epsilon^i - \frac{2h_I}{\sqrt{6}} \gamma^{\tilde{\rho}} F^I_{\tilde{\mu}\tilde{\rho}} \epsilon^i \\
\delta A^I_{\tilde{\mu}} &= -i h^I_a \bar{\epsilon}_i \gamma_{\tilde{\mu}} \lambda^{ai} - \frac{i\sqrt{6}}{2} h^I \bar{\Psi}_{\tilde{\mu} i} \epsilon^i \\
\delta \lambda^{ai} &= -f^a_x \gamma^{\tilde{\mu}} D_{\tilde{\mu}} \phi^x \epsilon^i - \frac{1}{2} h^a_I \gamma^{\tilde{\mu}\tilde{\nu}} \epsilon^i F^I_{\tilde{\mu}\tilde{\nu}} \\
\delta \phi^x &= -i f^x_a \bar{\epsilon}_i \lambda^{ai}
\end{aligned} \tag{8}$$

we see that the parity assignments are consistent if the supersymmetry parameters

$$\epsilon^1 = \begin{pmatrix} \varepsilon \\ \bar{\zeta} \end{pmatrix}, \ \epsilon^2 = \begin{pmatrix} \zeta \\ -\bar{\varepsilon} \end{pmatrix} \tag{9}$$

is taken to consist of even ε and odd ζ. Recall that ϵ^1, ϵ^2 are symplectic Majorana. It is easy to see that under ε transformations the fields of the Radion supermultiplet, $\left\{ \frac{1}{\sqrt{2}} h^0 e^{\dot{5}}_5 + i\sqrt{\frac{1}{3}} A^0_5, \quad h^0 \Psi^2_5 \right\}$ transform like a chiral multiplet, while the transformation of the even fields under ε-supersymmetry reads

$$\begin{aligned}
\delta e^m_\mu &= i \left(\varepsilon \sigma^m \bar{\psi}_\mu + \bar{\varepsilon} \bar{\sigma}^m \psi_\mu \right) \\
\delta \psi_\mu &= 2 \nabla_\mu(\omega) \varepsilon + \frac{i \ h_0}{2\sqrt{6}} \left[(\sigma_\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}_\mu) + 4 \delta^\nu_\mu \right] \varepsilon F^0_{\nu 5} \ e^5_{\dot{5}} + ... \\
\delta A^I_\mu &= -i h^I_a \left(\varepsilon \sigma_\mu \bar{\lambda}^a + \bar{\varepsilon} \bar{\sigma}_\mu \lambda^a \right) + ... \\
\delta \lambda^a &= -h^a_I \sigma^{\mu\nu} \varepsilon F^I_{\mu\nu} + i f^a_x \partial_5 \phi^x e^5_{\dot{5}} \varepsilon + ...
\end{aligned} \tag{10}$$

where $\psi_\mu \equiv \Psi^1_{\mu\, L}$ and $\lambda^a \equiv \lambda^{1a}_L$. The ellipsis in eq. (10) stand for even products of odd fields, and hence vanishing on the brane. Thus we see that on the branes determined

by the orbifold construction, we get the four-dimensional $N = 1$ on-shell transformation of the supergravity multiplet, the Yang-Mills vector multiplets and one chiral multiplet, the radion multiplet, surviving from the five-dimensional supergravity multiplet. Notice however the appearance of an extra $\partial_5 \phi^x$ term, and a $F^0_{\nu 5}$ dependent term in the gaugino and gravitino transformation laws respectively.

4 Bulk Gravity and Gauge Couplings of the Brane Multiplets

The matter fields are considered to be localized on the branes at the fixed points $x^5 = 0$ and $x^5 = \pi R$. For the purposes of this work it suffices to consider only the brane at $x^5 = 0$. The treatment of fields living on the brane at $x^5 = \pi R$ is done similarly.

The requirement of $N = 1$ local supersymmetry invariance on the branes determines the on-shell couplings of these fields to the gravity and gauge multiplets. These can be found following Nöther's procedure. This procedure is used in the on-shell formulation of local supersymmetry [29], where the rôle of the gauge field is played by the gravitino, while the gauge current is the supercurrent. However in the case of supersymmetry besides the modification of the Lagrangian the transformation laws should be also modified accordingly. This is well understood since the on-shell formulation follows from the off-shell after eliminating the auxiliary fields by solving the equations of motion which are modified upon changing the Lagrangian at each step.

The original Lagrangian is

$$\mathcal{L}_{\rm orig} = \mathcal{L}_0 + \mathcal{L}_b \tag{11}$$

with $\mathcal{L}_b$ the "brane" part including the interactions of the matter fields, localized on the brane, with the "projections" of the bulk fields, gravity and gauge fields, on the brane. The original SUSY transformations will be denoted by δ_0. $\mathcal{L}_0$ is invariant under δ_0, i.e. $\delta_0 \mathcal{L}_0 = 0$, but not $\mathcal{L}_b$ that is $\delta_0 \mathcal{L}_b \neq 0$. As already stated we must modify the original theory by adding new terms, $\Delta\mathcal{L}$, so that the total Lagrangian

$$\mathcal{L}_s = \mathcal{L}_0 + \mathcal{L}_b + \sum_k \Delta_k \mathcal{L}$$

is invariant under the modified SUSY transformations denoted by δ_s,

$$\delta_s = \delta_0 + \sum_k \delta_k$$

that is $\delta_s\mathcal{L}_s = 0$. We will proceed iteratively and in the above sums k denotes the iteration step.

In order to derive the gravitational couplings we ignore for the moment the gauge interactions and consider for simplicity just one chiral multiplet on the brane at $x^5 = 0$. Thus we start from $\mathcal{L}_b$ which for one chiral multiplet, (φ, χ), has the form [1]

$$\mathcal{L}_b = -e^{(5)}\Delta_{(5)}\left(\partial_\mu\varphi\partial^\mu\varphi^* + i\bar{\chi}\bar{\sigma}^\mu D_\mu\chi\right). \tag{12}$$

In order to facilitate the discussion we have ignored at this stage superpotential and gauge interactions. Since $\mathcal{L}_b$ includes dependencies on the vierbein it facilitates to write

$$\delta_0 = \delta_0^{(e)} + \delta_0^{(\mathrm{rest})} \tag{13}$$

with $\delta_0^{(e)}$, $\delta_0^{(\mathrm{rest})}$ denoting variations acting on the vierbein and the remaining fields respectively. With this we get from (12)

$$\delta_0\mathcal{L}_b = \delta_0^{(e)}\left[-e^{(5)}\Delta_{(5)}\left(\partial_\mu\varphi\partial^\mu\varphi^* + i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi\right)\right] + e^{(5)}\Delta_{(5)}\left(J_\mu\partial^\mu\varepsilon + h.c.\right) \tag{14}$$

where J^μ in (14) is the (Nöther) supercurrent given by $J^\mu = \sqrt{2}\chi\sigma^\mu\bar{\sigma}^\nu\partial_\nu\varphi^*$. According to Nöther's procedure in order to eliminate the last term in (14) we must add a term $\Delta_1\mathcal{L}$ while no change of SUSY transformations is required at this stage. Thus

$$\delta_1 = 0 \ \ , \ \ \Delta_1\mathcal{L} = -\frac{1}{2}e^{(5)}\Delta_{(5)}\left(J_\mu\psi^\mu + h.c.\right) \tag{15}$$

since in our conventions $\delta_0\psi_\mu \sim 2D_\mu\varepsilon + ...$ Next we have to check the invariance of the so constructed Lagrangian $\mathcal{L}_0 + \mathcal{L}_b + \Delta_1\mathcal{L}$ and modify it accordingly if it happens to be non-invariant under the new SUSY transformation law $\delta_s = \delta_0 + \delta_1$, which however at this stage, due to the vanishing of δ_1 coincides with the original transformation δ_0. Using the gravitino transformation law given by the second equation in eq. (10) one gets

$$\begin{aligned}
&\delta_s\left(\mathcal{L}_0 + \mathcal{L}_b + \Delta_1\mathcal{L}\right) = \\
&\delta_0^{(e)}\left[-e^{(5)}\Delta_{(5)}\left(\partial_\mu\varphi\partial^\mu\varphi^* + i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi\right)\right] - \frac{1}{2}e^{(5)}\Delta_{(5)}\left[\left(\delta_0 J_\mu\right)\psi^\mu + h.c\right] \\
&+\frac{1}{\sqrt{6}}e^{(5)}\Delta_{(5)}\, F^0_{\mu\dot{5}}\,\delta_0\left(J^{(\varphi)\,\mu} - \frac{1}{2}J^{(\chi)\,\mu}\right) + \delta_0^{(e)}\left(-\frac{1}{2}e^{(5)}\Delta_{(5)}J_\mu\psi^\mu + h.c.\right)
\end{aligned} \tag{16}$$

The third term in (16) follows from the second term of the gravitino transformation in (10), as can be verified by a straightforward algebra, and $J^{(\varphi)}$, $J^{(\chi)}$ denoting the $U_R(1)$ currents of φ and χ fields, given by

$$J_\mu^{(\varphi)} = -i\varphi^*\overset{\leftrightarrow}{\partial}_\mu\varphi\ , \quad J_\mu^{(\chi)} = \chi\sigma_\mu\bar{\chi} \tag{17}$$

[1] It is usefull to define $\Delta_{(5)}(x^5) \equiv \delta(x^5)/e_5^{\dot{5}} = e_{\dot{5}}^5\delta(x^5)$. Note that with the parity assignments we have adopted $e^{(5)}\,\Delta_{(5)} = e^{(4)}\,\delta(x^5)$ on the brane.

Note also that we have not included the spin connection $\omega_{\mu mn}$ for lack of space. Its contribution at each stage is determined by fully covariantizing the results.

The first three terms in (16) are cancelled if we modify the SUSY transformations as it appears below

$$\begin{aligned}\delta_2\,\varphi &= 0, \quad \delta_2\chi = -i\sigma^\mu\bar{\varepsilon}\,(\psi_\mu\chi) \\ \delta_2 e_\mu^m &= 0, \quad \delta_2\psi_\mu = \frac{i}{2}\Delta_{(5)}\left(J_\mu^{(\varphi)}\varepsilon - \sigma_{\mu\nu}\varepsilon J^{(\chi)\,\nu}\right)\end{aligned} \tag{18}$$

and add a term to the Lagrangian given by

$$\begin{aligned}\Delta_2\mathcal{L} &= e^{(5)}\Delta_{(5)}\Big\{\,\frac{1}{4}J_\sigma^{(\chi)}\left[\,iE^{\mu\nu\rho\sigma}\left(\psi_\mu\sigma_\nu\bar{\psi}_\rho\right) + \left(\psi_\mu\sigma^\sigma\bar{\psi}^\mu\right)\right] \\ &- \frac{i}{4}\,E^{\mu\nu\rho\sigma}J_\sigma^{(\varphi)}\psi_\mu\sigma_\nu\bar{\psi}_\rho - \frac{1}{\sqrt{6}}\Big(J^{(\varphi)\,\mu} - \frac{1}{2}J^{(\chi)\,\mu}\Big)F^0_{\mu\dot{5}}\,\Big\}\,,\end{aligned} \tag{19}$$

where $E^{\mu\nu\rho\sigma}$ is the four dimensional antisymmetric tensor. The last term in (19) is needed for the cancellation of the $F^0_{\mu\dot{5}}\left(-J^{(\varphi)\,\mu} + \cdots\right)$ term in (16). The need of introducing the remaining terms will be clarified in the following.

We next have to check the invariance of $\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L}+\Delta_2\mathcal{L}$ under δ_s transformations. Since $\delta_1 = 0$ we have

$$\delta_s(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L}+\Delta_2\mathcal{L}) = \delta_0(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L})+\delta_0(\Delta_2\mathcal{L})+\delta_2(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L}+\Delta_2\mathcal{L}) \tag{20}$$

As we have already discussed, from the transformation $\delta_0(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L})+\delta_0(\Delta_2\mathcal{L})$ only the term $\delta_0^{(e)}\left(\frac{e^{(5)}}{e_5^{\dot{5}}}J_\mu\psi^\mu + h.c.\right)$ survives. In fact the variation $\delta_0(\Delta_2\mathcal{L})$ is given by

$$\begin{aligned}\delta_0(\Delta_2\mathcal{L}) =& \frac{1}{\sqrt{6}}\left[-\delta_0\left(e^{(5)}\Delta_{(5)}F^0_{\mu\dot{5}}\right)\left(J^{(\varphi)\,\mu} - \frac{1}{2}J^{(\chi)\,\mu}\right) - e^{(5)}\Delta_{(5)}F^0_{\mu\dot{5}}\delta_0\Big(J^{(\varphi)\,\mu} - \frac{1}{2}J^{(\chi)\,\mu}\Big)\right] \\ &+ \delta_0^{(e)}\left\{\frac{1}{4}e^{(5)}\Delta_{(5)}\left[iE^{\mu\nu\rho\sigma}\left(\psi_\mu\sigma_\nu\bar{\psi}_\rho\right)\left(J_\sigma^{(\chi)} - J_\sigma^{(\varphi)}\right) + \left(\psi_\mu\sigma^\sigma\bar{\psi}^\mu\right)J_\sigma^{(\chi)}\right]\right\} \\ &+ \frac{1}{4}e^{(5)}\Delta_{(5)}\left[iE^{\mu\nu\rho\sigma}\left(\psi_\mu\sigma_\nu\bar{\psi}_\rho\right)\delta_0\left(J_\sigma^{(\chi)} - J_\sigma^{(\varphi)}\right) + \left(\psi_\mu\sigma^\sigma\bar{\psi}^\mu\right)\delta_0 J_\sigma^{(\chi)}\right] \\ &+ \frac{1}{4}e^{(5)}\Delta_{(5)}\left[iE^{\mu\nu\rho\sigma}\left(J_\sigma^{(\chi)} - J_\sigma^{(\varphi)}\right)\delta_0\left(\psi_\mu\sigma_\nu\bar{\psi}_\rho\right) + J_\sigma^{(\chi)}\delta_0\left(\psi_\mu\sigma^\sigma\bar{\psi}^\mu\right)\right]\end{aligned} \tag{21}$$

The term $\sim\ F^0_{\mu\dot{5}}\delta_0\left(J^{(\varphi)\,\mu} - \frac{1}{2}J^{(\chi)\,\mu}\right)$ in (21) cancels the corresponding term in eq. (16). Also the term in the last line cancels the first two terms of eq. (16) along with δ_2 variations of the gravitino ψ_μ and the fermion χ kinetic terms occuring within $\mathcal{L}_0+\mathcal{L}_b$, that is $\delta_2\left(\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu D_\rho\psi_\sigma - ie^{(5)}\Delta_{(5)}\bar{\chi}\bar{\sigma}^\mu D_\mu\chi\right)$. Actually that was the reason nonvanishing variations δ_2 had to be introduced for the χ and ψ_μ fields. However not all of the terms in

$\delta_s(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L}+\Delta_2\mathcal{L})$ are completely cancelled. Among those terms that survive is the δ_2 variation of $\Delta_1\mathcal{L}$ in (20) which reveals an interesting feature that needs be discussed. In fact

$$\delta_2(\Delta_1\mathcal{L}) = -\frac{1}{2}e^{(5)}\Delta_{(5)}J_\mu\delta_2\psi^\mu + \delta_2\left(-\frac{1}{2}e^{(5)}\Delta_{(5)}J_\mu\right)\psi^\mu + h.c. \tag{22}$$

and the first term in (22), after some straightforward algebra, is brought into the form

$$-\frac{i}{2\sqrt{2}}e^{(5)}{\Delta_{(5)}}^2\left[(\chi\sigma^\mu\bar{\sigma}^\nu\varepsilon)\,J^{(\varphi)}_\mu\partial_\nu\varphi^* - \frac{\sqrt{2}}{2}\chi\sigma^\mu\bar{\sigma}^\rho\sigma_{\mu\nu}\varepsilon J^{(\chi)\nu}\partial_\rho\varphi^*\right] + h.c. \tag{23}$$

due to the variation $\delta_2\psi_\mu$ (see eq. (18)). Since $i\sqrt{2}\bar{\sigma}^\nu\varepsilon\partial_\nu\varphi^*$ is actually $\delta_0\bar{\chi}$ we have from the expression (23) a contribution $-\frac{1}{4}e^{(5)}{\Delta_{(5)}}^2\chi\sigma^\mu\,(\delta_0\bar{\chi})\left(J^{(\varphi)}_\mu + \frac{1}{4}J^{(\chi)}_\mu\right) + h.c.$ Due to the appearance of this we need to add a new term in the Lagrangian which includes, among others, the aforementioned contribution that is

$$\Delta_3\mathcal{L} = \frac{1}{4}e^{(5)}{\Delta_{(5)}}^2J^{(\chi)}_\mu\left(J^{(\varphi)\,\mu} + \frac{1}{4}J^{(\chi)\,\mu}\right) + \cdots \tag{24}$$

In (24) the ellipsis denote additional terms. The terms in (24) are not new. In ref. [30] such terms do appear in the derived Lagrangian completing previous derivations, [31]. In that work the $\delta^2(x^5)$ terms complete a perfect square (see eq. (3.39) of that paper). This is not the case in our approach. However our results at this stage can not be directly compared to those of [30], due to the nontrivial conformal factor existing in eq. (3.39) of the aforementioned paper. The relation between the two approaches will be discussed later on.

The above results are easily extended in the case that the original brane action has the structure of a general σ-model ([32]),

$$\begin{aligned}\mathcal{L}_b = \; & - \; e^{(5)}\Delta_{(5)}\Big[K_{ij^*}D_\mu\varphi^iD^\mu\varphi^{*j} + (\frac{i}{2}\,K_{ij^*}\chi^i\sigma^\mu D_\mu\bar{\chi}^j \; + \; h.c) \\ & + \; \frac{1}{2}\,(D_iD_jW\;\chi^i\chi^j \; + \; h.c.) + \; K^{ij^*}D_iW\;D_{j^*}W^* - \frac{1}{4}R_{ij^*kl^*}\;\chi^i\chi^k\;\bar{\chi}^j\bar{\chi}^l\Big]\end{aligned} \tag{25}$$

where K_{ij^*} is the Kähler metric. In this equation $D_\mu\bar{\chi}^j$ is covariant under both spacetime and Kähler transformations. The superpotential and Yukawa terms, are also included. In the flat case $D_iW = \partial_iW$, $D_iD_jW = \partial_i\partial_jW - \Gamma^k_{ij}\,\partial_kW$. Later when considering the curved case it turns out that these include additional terms so that they are covariant with respect to the Kähler function K as well.

The coupling of the brane fields to the gauge and the gaugino fields propagating in the bulk is known from the flat case, see [16], so that here we will only outline the steps

we follow. As can be seen from (10) the transformation of λ^a, stemming from the five dimensions is not exactly that of a gaugino. The extra variation requires the addition to the Lagrangian of a term $g\ \Delta_{(5)}\ D^{(a)} f^a_x \partial_5 \phi^x$ while it is known that the variation of the gaugino-fermions Yukawa terms, given by $\Delta_{(5)}\ \left(-ig\sqrt{2} D^{(a)}{}_{,j^*}\ \bar{\chi}^j \bar{\lambda}^a + h.c. \right)$, requires the modification of the gaugino transformation rule by adding a term $\delta'_\varepsilon \lambda^a = -ig\Delta_{(5)}\ D^{(a)} \varepsilon$ and supersymmetry invariance is finally restored by adding the term

$$- \frac{g^2}{2} {\Delta_{(5)}}^2\ D^{(a)} D^{(a)}. \tag{26}$$

These are the generalizations of the results reached in [16] when the manifold of the scalar bulk fields is curved.

As far as the presence of the superpotential is concerned, we already know from the flat case it modifies the fermions supersymmetry transformation law according to

$$\delta'_\varepsilon \chi^i = -\sqrt{2}\ K^{ij^*} D_{j^*} W^* \varepsilon\,, \tag{27}$$

The extra variation of the fermion fields applied to the coupling of the Nöther current with the gravitino field $\sim\ J^\mu \psi_\mu$, see eq. (15), leads to modification of the gravitino transformation law as $\delta'_\varepsilon \psi_\mu = i\Delta_{(5)}\ W \sigma_\mu \bar{\varepsilon}\,,$ and the addition to the Lagrangian of the term

$$\mathcal{L}' = e^{(5)}\ \Delta_{(5)}\ \left[W^* \psi_\mu \sigma^{\mu\nu} \psi_\nu + W \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu \right]\ . \tag{28}$$

We can see in turn that its variation, due to $\delta'_\varepsilon \psi_\mu$ above and the supersymmetry transformation for $e^{(4)}$, see eq. (10), is

$$\delta'_\varepsilon \mathcal{L}' = -3\Delta_{(5)}\ \delta \left(e^{(5)} \Delta_{(5)}\ \right) \mid W \mid^2 \tag{29}$$

which is cancelled by the addition of the known $\mid W \mid^2$ term of the supergravity potential which however in our case it appears multiplied by ${\Delta_{(5)}}^2$. Variations of the potential terms $K^{ij^*} D_i W\ D_{j^*} W^*$ are cancelled by the Yukawa terms $\sim D_i D_j W \chi^i \chi^j + h.c.$ and those of the $\mid W \mid^2$ require the appearence of terms $\sim D_i W \chi^i \sigma^\mu \bar{\psi}_\mu + h.c.$ in the Lagrangian for their cancellation. This procedure can be continued and in the following steps the wellknown Kählerian exponents $e^{\,K/2}$ appearing in the ordinary 4 - D supergravity start showing up accompanying each power of the superpotential W, or derivative of it, both in the Lagrangian and the transformation laws. However the Kähler function K in the exponent appears multiplied by $\Delta_{(5)}\ =\ e^5_5\ \delta(x^5)$ as shown in the Lagrangian given below. In conjuction with this we point out that the covariant derivatives of the superpotential

W are also found to depend on the Kähler function through the combination $\Delta_{(5)}\, K$, rather than K itself, so that Kähler invariance is indeed maintained.

Summarizing, the interactions of a set of chiral multiplets localized on the brane designated by the index i, with the bulk gravity and gauge fields are found to be

$$
\begin{aligned}
\mathcal{L}^{(4)} = e^{(5)}\Delta_{(5)} \Big[& - K_{ij^*} D_\mu \varphi^i D^\mu \varphi^{*j} - (\frac{i}{2} K_{ij^*} \chi^i \sigma^\mu D_\mu \bar{\chi}^j + h.c) - ig\sqrt{2}(D^{(a)},_{j^*} \bar{\chi}^j \bar{\lambda}^a - h.c.) \\
& - \frac{g}{2} D^{(a)}(\psi_\mu \sigma^\mu \bar{\lambda}^a - \bar{\psi}_\mu \bar{\sigma}^\mu \lambda^a) - \frac{1}{\sqrt{2}} K_{ij^*}(D_\mu \varphi^{*j} \chi^i \sigma^\nu \bar{\sigma}^\mu \psi_\nu + D_\mu \varphi^i \bar{\chi}^j \bar{\sigma}^\nu \sigma^\mu \bar{\psi}_\nu) \\
& + \frac{i}{4} E^{\mu\nu\rho\sigma}(J_\sigma^{(\chi)} - J_\sigma^{(\varphi)}) \psi_\mu \sigma_\nu \bar{\psi}_\rho + \frac{1}{4} J_\sigma^{(\chi)} \psi_\mu \sigma^\sigma \bar{\psi}^\mu + \frac{1}{4} \Delta_{(5)}\, J^{(\chi)\,\mu}\, (J_\mu^{(\varphi)} + \frac{1}{4} J_\mu^{(\chi)}) \\
& - \frac{1}{8}\, R_{ij^*kl^*}\, \chi^i \sigma^\mu \bar{\chi}^j\, \chi^k \sigma_\mu \bar{\chi}^l\, - \frac{1}{4}\, (J_\mu^{(\varphi)} - \frac{1}{2} J_\mu^{(\chi)})\, \lambda^a \sigma^\mu \bar{\lambda}^a \\
& + \frac{1}{\sqrt{6}}\, (-J^{(\varphi)\,\mu} + \frac{1}{2} J^{(\chi)\,\mu})\, F^0_{\mu\dot{5}} - \frac{1}{2} g^2 \Delta_{(5)}\;\, D^{(a)} D^{(a)} + g D^{(a)} f_x^a \partial_5 \phi^x \\
& - e^{\Delta_{(5)}\, K/2}(W^* \psi_\mu \sigma^{\mu\nu} \psi_\nu + \frac{i}{\sqrt{2}} D_i W \chi^i \sigma^\mu \bar{\psi}_\mu + \frac{1}{2} D_i D_j W \chi^i \chi^j + h.c.) \\
& - e^{\Delta_{(5)}\, K}(K^{ij^*}\, D_i W\, D_{j^*} W^* - 3\Delta_{(5)}\, |W|^2) \Big] + \cdots
\end{aligned} \tag{30}
$$

where in the general case

$$
J_\mu^{(\varphi)} = -i \left(K_i \partial_\mu \varphi^i - K_{i^*} \partial_\mu \varphi^{*i} \right) \;, \quad J_\mu^{(\chi)} = K_{ij^*} \chi^i \sigma_\mu \bar{\chi}^j\,. \tag{31}
$$

The ellipsis in (30) stand for couplings of the brane fields with the radion multiplet, which is even, and other even combinations of odd fields which are not presented here. The prefactor $e^{(5)}\,\Delta_{(5)}$ in the Lagrangian above provides $e^{(4)}$ upon integration with respect x^5. Note that the terms $D^{(a)}D^{(a)}$, $J^{(x)}(\cdots)$, $|\,W\,|^2$ and the exponents involving the Kähler function appear multiplied by an extra power of $\Delta_{(5)}$ whose argument can be put to zero, due to the overall $\Delta_{(5)}$ multiplying the Lagrangian, which is proportional to $\delta(x^5)$. Since $\int_{-\pi R}^{\pi R} dx^5 e_5^{\dot{5}} \Delta_{(5)}(x^5) = 1$ and $\int_{-\pi R}^{\pi R} dx^5 e_5^{\dot{5}} = L$ is the "volume" of the fifth dimension, we are tempted to interpret $\Delta_{(5)}(0) \simeq 1/L \equiv M_L$. Replacing then $\Delta_{(5)}(0)$ by M_L and reestablishing units we find that M_L enters in our formulae only through the ratio M_5^3/M_L where M_5 is related to the 5 - D gravitational coupling through $k^2_{(5)} = 1/M_5^3$. The 4 - D gravitational constant is $k^2_{(4)} = k^2_{(5)}/L$ and the aforementioned ratio is related to the Planck scale via $M_5^3/M_L = M^2_{Planck}$. In doing all this the gravitino, gauge boson and gaugino, as well as the five dimensional gauge coupling should scale appropriately as $\psi_\mu = L^{-1/2} \hat{\psi}_\mu$, $A_\mu^{(a)} = L^{-1/2} \hat{A}_\mu^{(a)}$, $\lambda_\mu^{(a)} = L^{-1/2} \hat{\lambda}_\mu^{(a)}$ and $g = L^{1/2} g_{(4)}$, as dictated by the kinetic terms of these fields, in order for them to have the right normalization and the appropriate dimensions in four dimensions. It then turns that with this interpretation the

terms in (30) are exactly those encountered in the ordinary 4 - D supergravity involving the interactions of the chiral fields φ^i, χ^i among themselves and their interactions with the gravity and gauge multiplets. Exception to it are additional terms where bulk fields, involving $F^0_{\mu\dot{5}}, \partial_5\phi^x$, the radion multiplet etc., interact with the multiplets on the brane. This rather rough qualitative argument is only used to show the correctness of our results. In a decent mathematical way this can be seen after replacing the bulk fields which interact with the brane chiral multiplets by their classical equations of motion as was first done in the model studied in [16]. This is the case for instance with the D - terms which complete a perfect square, as in the flat case [16], involving the derivative $\partial_5\phi^x$. Eliminating this by its classical equation of motion results to the ordinary four dimensional D - terms [16,31]. The importance of the $\Delta_{(5)}(0)$ terms at the quantum level has been discussed in [16,30,31].

Before continuing two remarks are in order. In our approach we have not considered so far gauging of the R-symmetry of the five-dimensional supergravity and as a consequence there is no potential on the brane stemming from the coupling to the bulk fields [26, 27]. Also the cubic constraint (4) leads to a $D = 4$, $N = 1$, Yang-Mills supergravity with a gauge field kinetic function f_{ab} proportional to δ_{ab}.

Finally in order to make contact with the results reached in [30] we have to express the gravitational part in the unrescaled Weyl basis. This can be accomplished by performing a Weyl transformation in the four dimensional part of the metric $e^m_\mu = \omega\ \tilde{e}^m_\mu$ accompanied by the appropriate rescalings of the fermionic fields $\psi_\mu = \omega^{-1/2}\tilde{\psi}_\mu\ , \chi = \omega^{1/2}\ \tilde{\chi}$. In order to simplify the discussion we shall limit ourselves to the case of one chiral multiplet on the brane. Under this trasformation the gravity action becomes

$$-\frac{1}{2}e^{(5)}R^{(5)} = \tilde{e}^{(5)}\left(\frac{\Omega}{6}\ \tilde{R}^{(5)} - \frac{1}{4\Omega}\ \Omega,_\mu \Omega,^\mu + \cdots\right) \tag{32}$$

which coincides with the corresponding term in [30] if we chose

$$\omega^2 = -\frac{1}{3}\Omega\,, \qquad \Omega = -3 + \Delta_{(5)} \mid \varphi \mid^2 .$$

Note that in the above relation the non-trivial part of the Ω acts only on the brane. The second term in (32) combined with the scalar field kinetic term gives

$$-\Omega,_{\varphi\varphi^*}\partial_\mu\varphi\partial^\mu\varphi^* + \frac{1}{4\Omega}\ \hat{J}^{(\varphi)}_\mu \hat{J}^{(\varphi)\mu} \tag{33}$$

if the Kähler function K is related to Ω through $\Delta_{(5)}\ K = -3\ ln(-\frac{\Omega}{3})$. The hated currents $\hat{J}^{(\varphi)}_\mu, \hat{J}^{(\chi)}_\mu$ are defined as $\hat{J}^{(\varphi)}_\mu = -i(\Omega_\varphi\partial_\mu\varphi - h.c.)$ and $\hat{J}^{(\chi)}_\mu = \Omega_{\varphi\varphi^*}\tilde{\chi}\sigma_\mu\bar{\tilde{\chi}}$ respectively. Now in order to bring the fermion kinetic term to a form proportional to

$\Omega_{,\varphi\varphi^*}$, as in the scalars kinetic terms above, we have to shift the gravitino on the brane as $\tilde{\psi}_\mu = \hat{\psi} + i\,\frac{\Omega_{\varphi^*}}{\sqrt{2}\Omega}\,\sigma_\mu\,\bar{\tilde{\chi}}$. This except of putting the fermion kinetic term to its canonical form, in the above sense, produces a term $\tilde{e}^{(5)}\frac{1}{2\Omega}\hat{J}^{(\varphi)}_\mu\hat{J}^{(\chi)\,\mu}$. Such terms are also generated after the gravitino shift from the rescaled coupling of the Nöether current with the gravitino yielding $-\tilde{e}^{(5)}\frac{1}{\Omega}\hat{J}^{(\varphi)}_\mu\hat{J}^{(\chi)\,\mu}$. A third source of similar terms are those given by

$$-e^{(5)}\frac{i}{4}\Delta_{(5)}\left[\Delta_{(5)}K_{,\varphi\varphi^*}\left(K_{,\varphi}\,\partial_\mu\varphi - K_{,\varphi^*}\,\partial_\mu\varphi^*\right) - 2(K_{,\varphi\varphi\varphi^*}\,\partial_\mu\varphi - K_{,\varphi^*\varphi^*\varphi}\,\partial_\mu\varphi^*)\right]\chi\sigma^\mu\bar{\chi}$$

where the first stems from $\sim J^{(\varphi)\mu}J^{(\chi)}_\mu$ and the second from the Kähler covariantization of the fermion kinetic terms in (30). These terms after the rescaling yield $\tilde{e}^{(5)}\frac{1}{4\Omega}\hat{J}^{(\varphi)}_\mu\hat{J}^{(\chi)\mu}$. Adding these we find that their total contribution to the rescaled Lagrangian is $-\tilde{e}^{(5)}\frac{1}{4\Omega}\hat{J}^{(\varphi)}_\mu\hat{J}^{(\chi)\mu}$.

Finally from the $J^{(\chi)}_\mu J^{(\chi)\mu}$ and the Kähler curvature term in (30) we get, after performing the Weyl rescalings, $\tilde{e}^{(5)}\frac{1}{16\Omega}\hat{J}^{(\chi)}_\mu\hat{J}^{(\chi)\mu}$. Collecting then the $\hat{J}^{(\varphi)}\cdot\hat{J}^{(\chi)}$, $\hat{J}^{(\chi)}\cdot\hat{J}^{(\chi)}$ terms and the $\hat{J}^{(\varphi)}\cdot\hat{J}^{(\varphi)}$ term in (33) we arrive at the result

$$\tilde{e}^{(5)}\frac{1}{4\Omega}\left[\hat{J}^{(\varphi)} - \frac{1}{2}\hat{J}^{(\chi)}\right]^2$$

completing a perfect square of the matter currents. This term combined with those that are linear and bilinear in $F_{\mu\dot{5}}$ yields a perfect square given by

$$\tilde{e}^{(5)}\frac{3}{2\Omega}\left[\hat{F}_{\mu\dot{5}} - \frac{1}{\sqrt{6}}\left(\hat{J}^{(\varphi)}_\mu - \frac{1}{2}\hat{J}^{(\chi)}_\mu\right)\right]^2 \tag{34}$$

where $\hat{F}_{\mu\dot{5}} \equiv (-\Omega/3)F_{\mu5}\,e^5_{\dot{5}}$.

We therefore see that by a Weyl rescaling and collecting the necessary terms we are able to derive the Lagrangian terms given by eq. (3.39) in [30]. However in eq. (34) $\hat{F}_{\mu\dot{5}}$ appears instead of $F_{\mu\dot{5}}$ found in that reference. This apparently small difference has a rather major impact on the 4-D effective supergravity Lagrangian since one does not get a regular current-current interaction after integrating out the graviphoton field. The source of the discrepancy can be sought in the on-shell method employed in this work, in conjuction with the order, in the 5-D gravitational constant $k_{(5)}$, the couplings of the graviphoton to the brane fields have been derived. In our own frame and in the on-shell procedure except the existing $F^2_{\mu\dot{5}}$ kinetic term, which is of zeroth order, the interaction $F_{\mu\dot{5}}\,J^\mu$ was derived up to order $\mathcal{O}(k_{(5)})$. If we continue carrying out Noether's procedure higher order terms will be collected involving $F^2_{\mu\dot{5}}$ and $F_{\mu\dot{5}}\;J^\mu$. In order to reconcile (34) with the findings of ref. [30] these terms should exponentiate as $e^{\frac{2}{3}\,\Delta_{(5)}K}\,F^2_{\mu\dot{5}}$ and $e^{\frac{1}{3}\,\Delta_{(5)}K}\,F_{\mu\dot{5}}\;J^\mu$ respectively. Note that the exponent $\Delta_{(5)}K$ as well as the singular part of Ω are both of

order $\mathcal{O}(k_{(5)}^2)$. This may not be unfeasable. In fact exponentials of this sort, involving the Kähler function K, do indeed appear in the action of the supersymmetric sigma model when we localize the global supersymmetry. The appearance of these exponentials is equivalent to saying that the field $F_{\mu\dot{5}}$ is renormalized to $e^{\frac{1}{3}\Delta_{(5)}K}\, F_{\mu\dot{5}}$, or the same $(\,-3/\Omega\,)\, F_{\mu\dot{5}}$, having as effect the replacement of $\hat{F}_{\mu\dot{5}}$ in eq. (34) by $F_{\mu\dot{5}}$, obtaining thus complete agreement with [30] to all orders in $k_{(5)}^2$. Having in mind that the the coupling of $F_{\mu\dot{5}}$ to matter oughts to be of the form presented in [30], for it leads to the correct current-current interaction in the effective 4-D Lagrangian, we argue that Noether's procedure oughts to yield the above renormalization for $F_{\mu\dot{5}}$ in the sense outlined previously. In order to check if this is indeed the case higher order interactions, in the gravitational constant $k_{(5)}$, of the brane fields with the five-dimensional gravity multiplet have to be derived in the on-shell scheme we have adopted. This rather complicated task, along with the derivation of additional terms coupling the brane fields to the radion multiplet, and other even combination of odd fields, which complete the Lagrangian given by (30), will appear in a future publication.

5 Discussion

In the context of $D = 5$, $N = 2$, Yang-Mills Supergavity compactified on S^1/Z_2 we consider the supersymmetric coupling of matter fields propagating on the brane at $x^5 = 0$. Working in the on-shell scheme we have derived the terms of the brane action which are relevant for studying the mechanisms of supersymmetry and gauge symmetry breaking. The omitted radion multiplet couplings, as well as other couplings to the brane fields, can be derived, if desired, using the Nöther procedure which we followed in this paper. The complete brane action including these terms and the mechanisms of supersymmetry and the gauge symmetry breaking in particular unified models, in which both Gravity and Gauge forces propagate in the bulk, will be the issue of a forthcoming publication.

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